PHYS 101: General Physics 1 KOÇ UNIVERSITY Fall Semester 2011

College of Sciences

Section 1 Quiz 13 28 December 2011

Closed book. No calculators are to be used for this quiz.
Quiz duration: 15 minutes

Name: Student ID: Signature:
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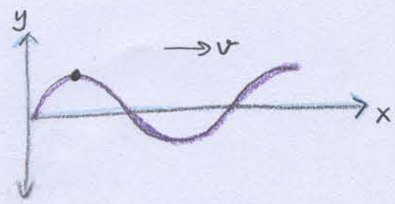
A transverse wave described by the wavefunction $y(x,t) = A\cos(kx + \omega t)$ is traveling on a string.

a) What is the longitudinal speed of a point on the string?
b) What is the maximum speed of the same point on the string?
c) Under what circumstances is the result of part (b) equal to the propagation speed, v, of the wave?

a)



Longitudinal speed of a point is zero. Since points move only in the transverse direction.

b) From the wave function we can get an expression for the transverse velocity of some point on the string in a transverse wave. v_y is different from the wave propogation speed v.

$$\Rightarrow \quad y(x,t) = A\cos(kx+\omega t)$$

To find the v_y at some point, we take derivative of the wave function $y(x,t)$ with respect to t, keeping x constant.

$$\Rightarrow \quad v_y(x,t) = \frac{\partial y(x,t)}{\partial t} = -\omega A \sin(kx+\omega t)$$

$$-1 \leq \sin(kx+\omega t) \leq 1$$

$\Rightarrow$ The maximum speed of the same point on the string is

$$v_y(x,t) = -\omega A.(-1) = \omega A$$

c) $v = \omega A \Rightarrow v = (vk).A \Rightarrow A = \frac{1}{k} = \frac{\lambda}{2\pi}$

$$A = \frac{\lambda}{2\pi}$$

Under this condition maximum speed of a point is equal to the propogation speed.

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Section 2 **Quiz 13** **28 December 2011**

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Name: **Student ID:** **Signature:**

A string of length 4m and of weight 1N is tied to the ceiling at one end. A block of weight 15N is suspended from the lower end. The string is plucked slightly close to the upper end and waves described by the wave function $y(x,t) = A\cos(kx - \omega t)$ travels down the rope, where $A = 10\text{cm}$. At any instant there are 50 wavelengths fitting the length of the rope.

a) Find the wavenumber of the wave.
b) Find the frequency of the wave.

This problem is similar to Exomple 15.3, page: 486

The tension of the string due to the block is 15 N and the string's linear mass density is

$$\mu = \frac{m_{string}}{L} \quad , \quad m_{string} = \frac{W_{string}}{g} = \frac{1N}{10\ m/s^2} = \frac{1}{10}\ kg$$

$$\mu = \frac{1/10\ kg}{4m} = \frac{1}{40}\ kg/m$$

a) $\frac{L}{\lambda} = 50 \Rightarrow \lambda = \frac{L}{50} = \frac{4}{50}\ m$

$$k : \text{wavenumber} = \frac{2\pi}{\lambda} = 25\pi$$

b) $v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{15N}{1/40\ kg/m}} = 10\sqrt{6}\ m/s$

$$v = \lambda \cdot f \Rightarrow f = \frac{v}{\lambda} = \frac{10\sqrt{6}}{4/50} = 125\sqrt{6}\ s^{-1}$$

We did not take into account the effect of the mass of the string on the tension. Because of the string's weight, its tension is greater at the top than at the bottom. Hence the wave speed increases as a wave travels up the rope:

$\Rightarrow F = 16\ N \qquad v = \sqrt{\frac{16N}{1/40\ kg/m}} = 8\sqrt{10}\ m/s$ at the top

$$\Rightarrow f = \frac{v}{\lambda} = 100\sqrt{10}\ s^{-1}$$

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Section 3 **Quiz 13** **28 December 2011**

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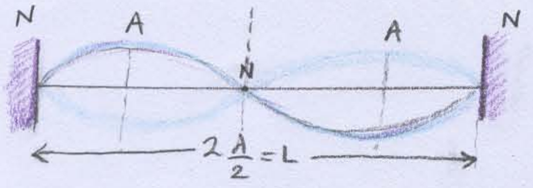
Name: **Student ID:** **Signature:**

A wire with mass m lenght L is fixed at both ends. The wire vibrates in its second harmonic frequency, f, with an amplitude A at the antinodes.

a) Sketch the standing wave formed on the wire at this frequency. Show the antinodes in your sketch clearly.
b) What is the speed of propagation of transverse waves in the wire?
c) Find the tension in the wire.

a)



$n=2$: second harmonics
f_2 (first overtone)

N: nodes
A: antinodes

b) $L = \frac{2\lambda}{2} \Rightarrow \lambda = L$

$v = \lambda . f \Rightarrow v = L . f$, speed of propogation of transverse waves in the wire.

c) $v = \sqrt{\frac{F}{\mu}}$, $\mu = \frac{m}{L}$

$\Rightarrow F = v^2 . \mu$

$\Rightarrow F = L^2 . f^2 . \frac{m}{L}$

$\Rightarrow F = L f^2 m$, tension in the wire

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Section 4 **Quiz 13** **28 December 2011**

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Name: **Student ID:** **Signature:**

Show that the average power of a wave on a string can also be written as $P_{av} = \frac{1}{2}Fk\omega A^2$, where F is the tension in the string, k is the wavenumber, ω is the angular frequency and A is the amplitude. If the tension in the string is quadrupled (i.e. F becomes 4F) while the amplitude is kept constant, how must k and ω each change to keep the average power constant?

a) $P_{av} = \frac{1}{2}\sqrt{\mu F}\,\omega^2 A^2$ Eq. 15.25

$v = \sqrt{\frac{F}{\mu}} \Rightarrow \sqrt{\mu} = \frac{1}{v}\sqrt{F}$

Then,

$P_{av} = \frac{1}{2}\frac{1}{v}\sqrt{F}\sqrt{F}\,\omega^2A^2 = \frac{1}{2}\frac{1}{v}F\omega^2A^2$

$\omega = vk$ Eq. 15.6 So, $\frac{1}{v} = \frac{k}{\omega}$

$P_{av} = \frac{1}{2}\frac{k}{\omega}F\omega^2A^2 = \frac{1}{2}Fk\omega A^2$ (1)

b) For the ω dependence, use Eq. (15.25), since it involves just ω

P_{av}, μ, A are constants, so $\sqrt{F}\omega^2$ is constant

$F_2 = 4F_1$

$\Rightarrow \sqrt{F_1}\,\omega_1^2 = \sqrt{F_2}\,\omega_2^2$

$\Rightarrow \sqrt{F_1}\,\omega_1^2 = \sqrt{4F_1}\,\omega_2^2$

$\Rightarrow \omega_2 = \frac{\omega_1}{\sqrt{2}}$

For k dependence, use Eq. (1); Then, P_{av} and A are constant with

$F_1k_1\omega_1 = F_2k_2\omega_2$

$\Rightarrow F_1k_1\omega_1 = 4F_1k_2\frac{\omega_1}{\sqrt{2}}$ $\Rightarrow k_2 = \frac{\sqrt{2}}{4}k_1$ to keep the average power constant.

2nd solution

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Section 4 **Quiz 13** **28 December 2011**

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Name: **Student ID:** **Signature:**

Show that the average power of a wave on a string can also be written as $P_{av} = \frac{1}{2}Fk\omega A^2$, where F is the tension in the string, k is the wavenumber, ω is the angular frequency and A is the amplitude. If the tension in the string is quadrupled (i.e. F becomes 4F) while the amplitude is kept constant, how must k and ω each change to keep the average power constant?

$$P(x,t) = F_y(x,t)\,v_y(x,t) = -F\frac{\partial y(x,t)}{\partial x}\frac{\partial y(x,t)}{\partial t} \qquad (15.21)$$

For a sinusoidal wave, the wave function is given by

$$y(x,t) = A\cos(kx-wt)$$

$$\frac{\partial y(x,t)}{\partial x} = -kA\sin(kx-wt) \quad , \quad \frac{\partial y(x,t)}{\partial t} = wA\sin(kx-wt)$$

$$P(x,t) = FkwA^2\sin^2(kx-wt) \qquad (15.22) \longrightarrow \text{instantaneous power}$$

$$0 \le \sin^2(kx-wt) \le 1$$

$$\Rightarrow P_{max} = FkwA^2$$

Let's take the average value of $P(x,t)$ over any whole number of cycles = over one period.

$$\boxed{P_{ave} = \frac{1}{T}\int_0^T P(x,t)\,dt} = \frac{1}{T}FkwA^2 \overbrace{\int_0^T \sin^2(kx-wt)\,dt}^{\text{Integral}}$$

$$I = \int_0^T \sin^2(kx-wt)\,dt = \int_0^T\left(\frac{1-\cos 2(kx-wt)}{2}\right)dt = \frac{1}{2}\int_0^T dt - \frac{1}{2}\int_0^T \cos 2(kx-wt)\,dt$$

$$I = \frac{1}{2}T - \frac{1}{2}\left(\frac{\sin 2(kx-wt)}{-2w}\Big|_0^T\right) = \frac{T}{2} + \frac{1}{4w}\left[\sin 2(kx - \overset{w}{\widetilde{\tfrac{2\pi}{T}}}\cdot T) - \sin 2(kx - \overset{w}{\widetilde{\tfrac{2\pi}{T}}}\cdot 0)\right]$$

$$\sin 2(kx \pm 2\pi) = \sin 2(kx)$$

$$\Rightarrow I = \frac{T}{2} \quad \Rightarrow \quad P_{ave} = \frac{1}{T}\cdot FkwA^2\cdot\frac{T}{2} = \frac{1}{2}FkwA^2 = \frac{P_{max}}{2}$$

where we applied $\boxed{\cos 2u = 1 - 2\sin^2 u}$

$$\text{If } F' = 4F \quad \Rightarrow v' = \sqrt{\frac{4F}{\mu}} \quad \Rightarrow v = \sqrt{\frac{F}{\mu}} \quad \Rightarrow v' = 2v \quad , \; w = vk$$

$$w' = v'k' \quad \Rightarrow \frac{w'}{k'} = 2v = 2\frac{w}{k}$$

$$\text{constant} = P_{av} = \frac{1}{2}F'k'w'A^2 = \frac{1}{2}(4)F\cdot \underbrace{k'w'}_{(1/4)kw}\cdot A^2 \quad \Rightarrow \overset{(1)}{\boxed{k'\cdot w' = \frac{1}{4}kw}} \quad \& \quad \overset{(2)}{\boxed{\frac{w'}{k'} = 2\frac{w}{k}}} \qquad \boxed{k' = \frac{\sqrt{2}}{4}k}$$

From (1) $k' = \frac{1}{4w'}kw \longrightarrow$ put in (2) $\Rightarrow \frac{w'\cdot 4w'}{kw} = \frac{2w}{k} \Rightarrow \boxed{w' = \frac{1}{\sqrt{2}}w} \longrightarrow$ put (1) $\Rightarrow k'\frac{1}{\sqrt{2}}w = \frac{1}{4}kw$

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Section 5 **Quiz 13** **28 December 2011**

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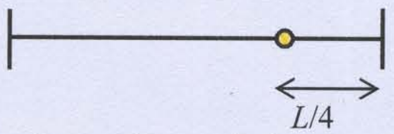
Quiz duration: 15 minutes

Name: **Student ID:** **Signature:**

A tight string of mass m length L is fixed at both ends. The tension in the string is F. A small bead is fixed on the string at a distance L/4 from the right end.

a) What is the smallest frequency of the waves that can be generated on the string under the condition that the bead should not move at all?

b) What is the smallest frequency of the waves that can be generated on the string under the condition that the bead should have maximum transverse displacement?

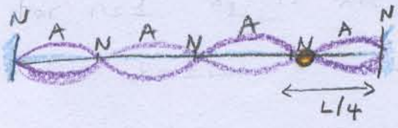


a) If there appears a node at a distance L/4 from the right end, the bead does not move. For smallest frequency, λ should be maximum. Since, $v = \lambda . f$

For the normal modes of the string, we have

$$\lambda_n = \frac{2L}{n} \quad (15.31)$$

max $n \to$ min



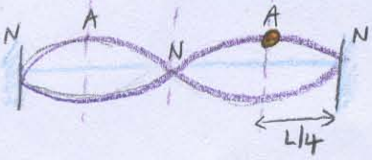
For the possible maximum λ, we have $n=4$: fourth harmonic, f_4 (third overtone)

$$\lambda_4 = \frac{2L}{4} = \frac{L}{2}$$

$v = \sqrt{\frac{F}{\mu}}$, $\mu = \frac{m}{L}$ $\Rightarrow v = \sqrt{\frac{FL}{m}}$

$\Rightarrow v = \lambda_4 . f_4$ $\Rightarrow f_4 = \frac{v}{\lambda_4} = \frac{\sqrt{\frac{FL}{m}}}{\frac{L}{2}} = 2 . \sqrt{\frac{F}{mL}}$, possible minimum frequency

b) The maximum transverce displacement occurs at antinodes



For possible maximum λ, we get $n=2$: second harmonic, f_2 (first overtone)

$\lambda_2 = \frac{2L}{2} = L$ $v = \sqrt{\frac{FL}{m}}$ $f_2 = \frac{v}{\lambda_2} = \frac{\sqrt{\frac{FL}{m}}}{L} = \sqrt{\frac{F}{mL}}$, possible minimum frequency